inmos

complex (I&Q) processing with the IMS A100

11.1 Introduction

Complex processing, involving in-phase and quadrature signal components, is necessary in many signal processing applications. This type of processing is needed in cases where the phase of the signal has significant impact on the processing outcome. For example consider a simple demodulator as shown in figure 11.1. The incoming tone, ($A \cos \omega t$), is demodulated by mixing it with a local oscillator having the same frequency. The output of the mixer is low-pass filtered to yield the final result. If there is a phase difference Φ between the incoming tone and the local oscillator signal, the output would be proportional to $\cos \Phi$. This indicates that the output of our simple demodulator is strongly dependent on the relative phases of the incoming and local oscillator signals. For the worst case of $\Phi = \frac{\pi}{2}$, the output would become zero! This means that the relative phase shift of the local oscillator can be quite disasterous.

Let us now perform the same demodulation using complex processing. The input signal can be represented in its complex form consisting of real and imaginary parts i.e.

$$x(t) = Ae^{-j\omega t} = A[\cos(\omega t) - j \sin(\omega t)] \tag{1}$$

Mixing the signal with a complex version of our local oscillator signal i.e. $A^{j(\omega t+\Phi)} = \cos(\omega t + \Phi) + j \sin(\omega t + \Phi)$ yields:

$$Ae^{-j\omega t} e^{j(\omega t+\Phi)} = Ae^{j\Phi} = A \cos \Phi + jA \sin \Phi \tag{2}$$

Note that this output is complex and contains both phase (Φ) and amptitude (A) information. The amptitude can be extracted by taking the modulus of the output i.e.

$$amplitude = \sqrt{(A \cos \Phi)^2 + (A \sin \Phi)^2} = |A| \tag{3}$$

The above example illustrates how complex processing can be used to preserve both phase and amptitude information in a simple demodulator.

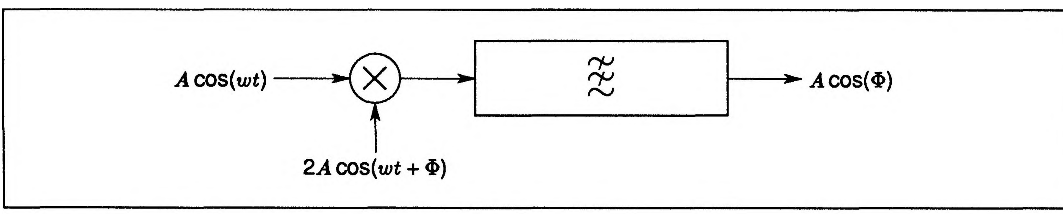


Figure 11.1 Simple demodulator

Similar phase related problems arise in correlation and convolution evaluations where complex processing becomes necessary for preserving the integrity of signals. This application note describes how on-chip facilities of the IMS A100 transversal filter can be used to perform complex correlation, convolution and filtering.

As described in the data sheet, the IMS A100 transversal filter incorporates two sets of coefficient memories (figure 11.2), each containing 32 16-bit words. At any instant one set of coefficients is applied to the multiply-accumulate array, whilst the other set can be accessed via the IMS A100 standard memory interface. The function of the two memory banks can be interchanged by performing a write operation to the 'Bank Swap' bit of a control register.

This allows the new set of coefficients to be used in the computation at the beginning of the next cycle. In this operation once the two memory banks are interchanged, the 'Banks Swap' control bit is reset by the device. No more interchanges are performed unless the bank swap control bit is again set by the host.

There is another control bit in the static control register of the IMS A100, that when set continuously interchanges the two memory banks at the beginning of each and every computation cycle. When this mode is set, alternate coefficient memory banks will be used for even and odd computation cycles. This mode is particularly suitable for implementing complex data processing. The following two sections describe how this continuous-swap mode can be employed to perform complex convolutions and correlations using the

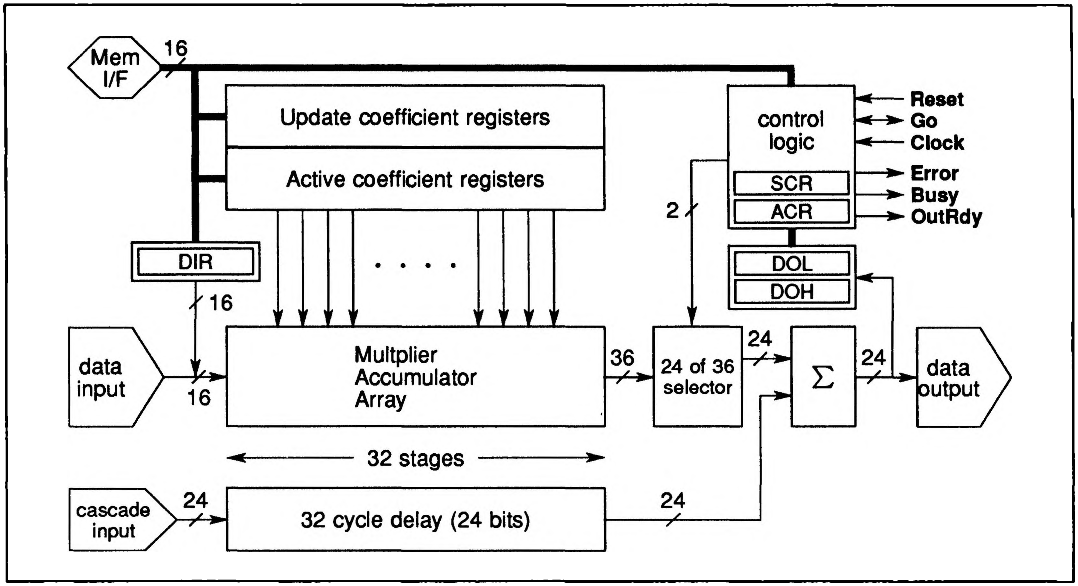


Figure 11.2 User's model of the IMS A100

IMS A100 transversal filters. A separate application note, available from INMOS, deals with the correlation and convolution concepts and their implementation using the IMS A100 device. Readers unfamiliar with the IMS A100 and its implementation of correlation and convolution functions are advised to refer to that application note before reading the following sections.

11.2 Complex correlation

The complex correlation between two signals r and s is very similar to real correlation (refer to the application note entitled 'Correlation and convolution with the IMS A100') with the difference that one of the two signals has to be complex conjugated first i.e.

$$R_{rs}(m) = \frac{1}{N}\sum_{k=0}^{N-1} r^*(k)s(k+m) \tag{3}$$

or

$$R_{rs}(m) = \frac{1}{N}\sum_{k=0}^{N-1} r(k)s^*(k+m) \tag{4}$$

where * indicated complex conjugate operation and both waveforms r and s can be complex.

Let us now investigate how the IMS A100 can perform this function. As shown in figure 11.2, the computational core of the IMS A100 contains an array of 32 multiply-and accumulators. In order to simplify the explanation of complex processing, let us consider a simple five-stage transversal filter as shown in figure 11.3. Once you have understood how such a simple structure can be used for complex correlation, it should be easy to extend the idea to larger correlations sizes involving one or many cascaded IMS A100 devices.

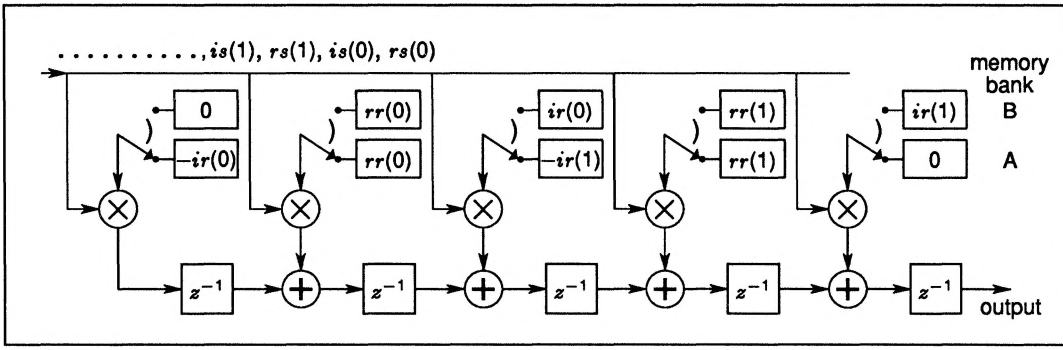


Figure 11.3 Two-point complex correlator based on the IMS A100 architecture

Suppose we want to perform a two-point complex correlation between a reference signal and a sequence of complex input samples. Let us denote the two complex samples of the reference signal with

$$r(0) = rr(0) + j\ ir(0) \quad and \quad r(1) = rr(1) + j\ ir(1)$$

where rr and ir indicate real parts and imaginary parts of the reference signal respectively.

Assume the input sequence is the following set of complex samples:

$$s(0) = rs(0) + j\ is(0),\ s(1) = rs(1) + j\ is(1),\ \ldots\ldots\ldots\ldots, s(n) = rs(n) + j\ is(n), \ldots\ldots\ldots$$

where $rs(n)$ and $is(n)$ indicate real and imaginary parts of the nth input sample, $s(n)$, respectively.

The 5-stage transversal filter shown in figure 11.3 can be used to correlate these two sequences. The reference signal samples are first complex conjugated i.e.

$$r^*(0) = rr(0) - j\ ir(0) \quad and \quad r^*(1) = rr(1) - j\ ir(1).$$

These samples of r^* are then allocated to the two coefficient memory banks as shown in figure 11.3. It can be seen from this diagram that both coefficient stores contain real and imaginary samples of the reference signal.

Assume that the input sequence is sampled into the correlator, with the real part followed by the imaginery part of each input sample. i.e. the input to the correlator is

$$rs(0),\ is(0),\ rs(1),\ is(1)\ , rs(2)\ , is(2)\ \ldots\ldots\ldots\ldots\ldots$$

where $rs(0)$ is the first input sample. Also assume that we have selected the continuous-swap mode so as the memory banks A and B are swapped every time a new input is sampled. (On the IMS A100, you can select this mode by writing to a control register). Assuming that the coefficient bank 'A' is selected for the first input sample, B for the second and so on, you should be able to convince yourself that the output sequence for the arrangement in figure 11.3 is as shown in table 11.1. Note that in this example it is assumed that the correlator is cleared first by writing several zero's to the input.

Sample number	Input sample	Output sample value		
1	$rs(0)$	0	$\Longrightarrow$	0
2	$is(0)$	$rs(0) \times rr(1) + is(0) \times ir(1)$	$\Longrightarrow$	Real part of $s(0) \times r^*(1)$
3	$rs(1)$	$-rs(0) \times ir(1) + is(0) \times rr(1)$	$\Longrightarrow$	Imag. part of $s(0) \times r^*(1)$
4	$is(1)$	$rs(0) \times rr(0) + is(0) \times ir(0) +$ $rs(1) \times rr(1) + is(1) \times ir(1)$	$\Longrightarrow$	Real part of $s(0) \times r^*(0) + s(1) \times r(1)$
5	$rs(2)$	$-rs(0) \times ir(0) + is(0) \times rr(0)$ $-rs(1) \times ir(1) + is(1) \times rr(1)$	$\Longrightarrow$	Imag. part of $s(0) \times r^*(0) + s(1) \times r^*(1)$
6	$is(2)$	$rs(1) \times rr(0) + is(1) \times ir(0) +$ $rs(2) \times rr(1) + is(2) \times ir(1)$	$\Longrightarrow$	Real part of $s(1) \times r^*(0) + s(2) \times r(1)$
7	$rs(3)$	$-rs(1) \times ir(0) + is(1) \times rr(0)$ $-rs(2) \times ir(1) + is(2) \times rr(1)$	$\Longrightarrow$	Imag. part of $s(1) \times r^*(0) + s(2) \times r^*(1)$
8	$is(3)$	$rs(2) \times rr(0) + is(2) \times ir(0) +$ $rs(3) \times rr(1) + is(3) \times ir(1)$	$\Longrightarrow$	Real part of $s(2) \times r^*(0) + s(3) \times r(1)$
9	$rs(4)$	$-rs(2) \times ir(0) + is(2) \times rr(0)$ $-rs(3) \times ir(1) + is(3) \times rr(1)$	$\Longrightarrow$	Imag. part of $s(2) \times r^*(0) + s(3) \times r^*(1)$
10	$is(4)$	$rs(3) \times rr(0) + is(3) \times ir(0) +$ $rs(4) \times rr(1) + is(4) \times ir(1)$	$\Longrightarrow$	Real part of $s(3) \times r^*(0) + s(4) \times r(1)$
11	$rs(5)$	$-rs(3) \times ir(0) + is(3) \times rr(0)$ $-rs(4) \times ir(1) + is(4) \times rr(1)$	$\Longrightarrow$	Imag. part of $s(3) \times r^*(0) + s(4) \times r^*(1)$

Table 11.1 Output sequence for figure 11.3

The last column in table 1 expresses the output sequence in terms of the complex input and complex reference samples. Examination of the output sequence would indicate that alternate samples correspond to real and imaginary parts of the expected correlation function. The arrangement for the two point correlator of figure 11.3, can be generalised to N-point complex correlation. Figure 11.4 illustrates the allocation of a reference signal to the coefficient memories of the IMS A100 for a 15-point complex correlation. The 15 complex samples of the reference signal are represented by:

$$r(n) = rr(n) + j\ ir(n) \qquad for \quad n = 0 \rightarrow 14$$

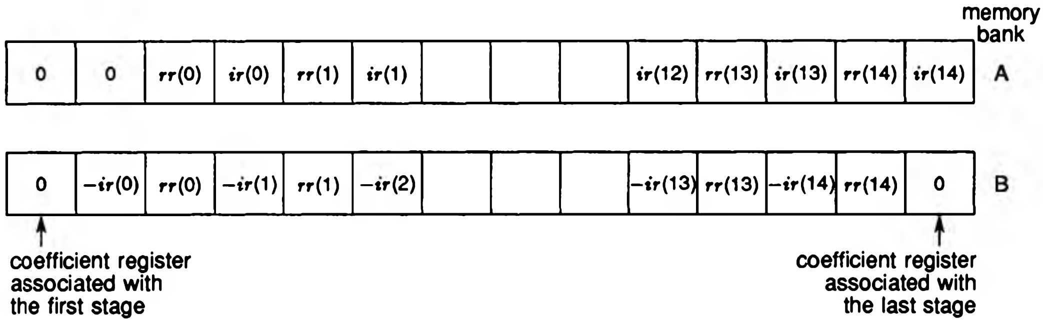
where $rr(n)$ and $ir(n)$ are the real and imaginery parts of the nth sample of the reference waveform. Similar to the 2-point complex correlator described earlier, the correct operation is achieved if each input sample is supplied to the chip with its real parts followed by its imaginery parts. The coefficient memories, of course, should be set to the continuous-swap mode.

In general for an N-point correlator realized with the IMS A100 chip, the first sample will always be zero (see table 1). The following $N-1$ output-sample pairs (real and imaginary parts) correspond to partial results for the following complex correlation coefficients:

$$R_{sr}(-(N-1)),\ R_{sr}(-(N-2)),\ \ldots\ldots\ldots\ldots,\ Rsr(-1)$$

and these will be followed with fully formed correlation coefficients:

$$R_{sr}(0),\ R_{sr}(1),\ Rsr(2)\ldots\ldots\ldots\ldots$$



The reference signal is defined by $r(n) = rr(n) + j\,ir(n)$ $\quad n = 0$ to 14.

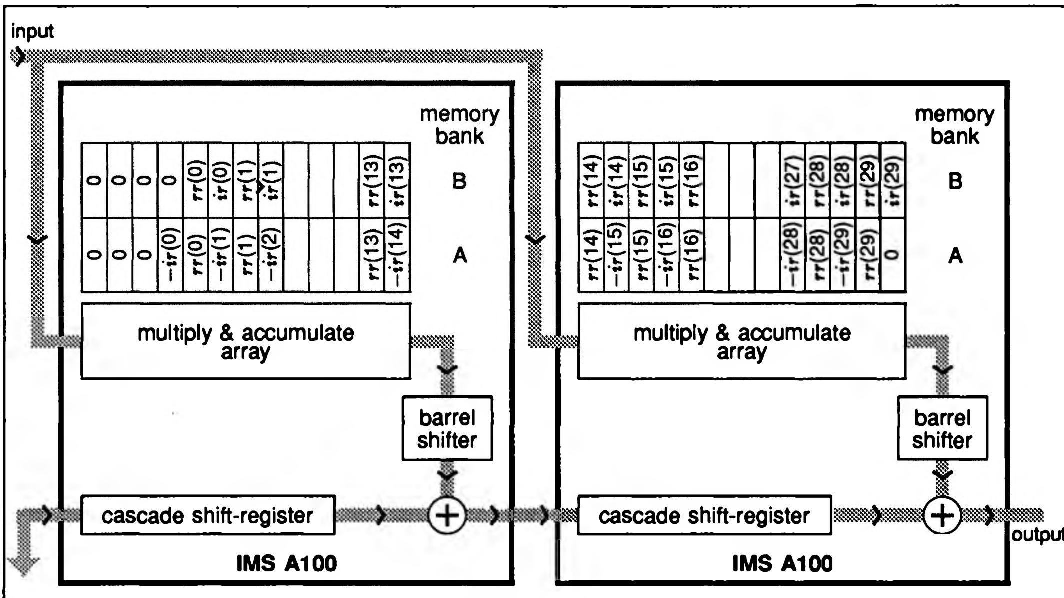
Each sample of the input signal is supplied with the real part followed by the imaginary part.

Starting with coefficient set A, the banks are automatically swapped for each write to the input device.

Figure 11.4 Example of reference signal allocation for a 15 point complex correlation using the IMS A100

Complex correlators involving more than 15-points can be implemented either by cascading several IMS A100 devices or alternatively by using mathematical decomposition techniques to convert a long correlation into several short ones which can then be evaluated using a single device. Although the latter approach would require fewer devices, the processing rate would be less than the cascade arrangement.

The IMS A100 can be cascaded without any external components to achieve correlators involving large number of correlation points. As an example, figure 11.5 illustrates how a 31-point complex correlator can be made up by cascading two IMS A100 devices. The allocation of a complex 30-point reference signal to the coefficient memories is also shown in figure 11.5. The input sequence, having a format described earlier, is supplied to both devices.



Allocation of the reference signal samples for a 30 point complex correllation is illustrated.

Figure 11.5 Cascading two IMS A100 devices to obtain a 31 point complex correlator

11.3 Complex convolution

The convolution process is closely related to that of correlation. In order to convolve two signals, one of the signals is time reversed and the second signal is then correlated (without complex conjugate operation) with this time reversed waveform i.e.

$$C_{rs}(m) = \frac{1}{N}\sum_{k=0}^{N-1} r(k)s(m-k). \tag{5}$$

The process of convolution is what happens in filters where the output corresponds to a convolution of the input signal and the impulse response of the filter. This is equivalent to correlating (without conjugate operation) time-reversed version of the impulse response with the input sequence.

The IMS A100 transversal filter can be used to perform complex convolution between a reference signal

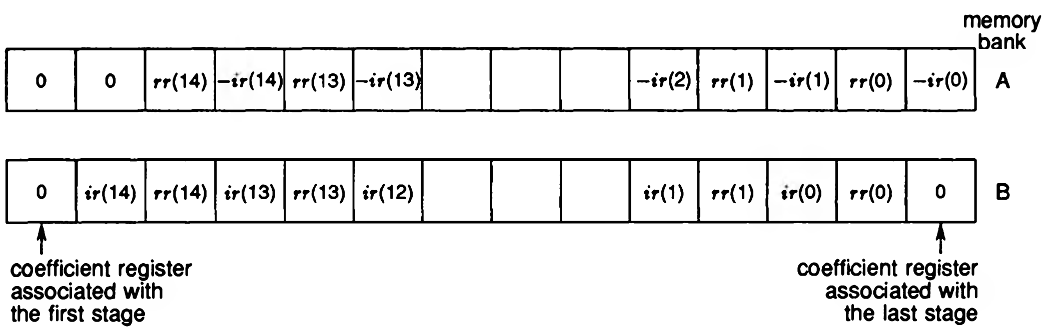
$$r(n) = rr(n) + jir(n) \qquad for \quad n = 0 \rightarrow N-1$$

and an input sequence

$$s(0) = rs(0) + j\ is(0),\ \ s(1) = rs(1) + j\ is(1),\ \ ....,\ s(n) = rs(n) + j\ is(n), ...$$

Figure 11.6 illustrates how the samples of a reference signal should be loaded in the coefficient memories for a 15-point complex convolution. In a similar fashion to the complex correlator implementation, the waveform to be convolved with this reference is applied to the input of the IMS A100 with the real part followed by the imaginary part. The coefficient memory banks should be set to the continuous bank-swap mode as before.

Again several IMS A100 devices can be cascaded to implement longer complex convolvers.



The reference signal is defined by $r(n) = rr(n) + jir(n)$ $\quad n = 0$ to 14.

Each sample of the input signal is supplied with the real part followed by the imaginary part.

Starting with coefficient set A, the banks are automatically swapped for each write to the input device.

Figure 11.6 Example of reference signal allocation for a 15 point complex convolution (filtering) using IMS A100